Inference About Two Samples

Independent samples (Section 9.3) Alternative method
Requirements: The two samples are independent. Both samples are simple random samples. Either of these conditions is satisfied: The two samples are large (with \( n_1 > 30 \) and \( n_2 > 30 \)) or both samples come from populations having normal distributions.

Test Statistic for unequal variances assumption
Rule of thumb, use when \( \frac{s_{\text{largest}}^2}{s_{\text{smallest}}^2} > 3 \)

\[
t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}
\]

Degrees of freedom: Conservative estimate: smaller of \( n_1-1 \) or \( n_2-1 \)

Test Statistic for equal variances assumption
Rule of thumb, use when \( \frac{s_{\text{largest}}^2}{s_{\text{smallest}}^2} \leq 3 \)

\[
t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}
\]

Degrees of freedom: \( n_1+n_2-2 \)

Where \( s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} \)

Dependent samples or matched pairs (Section 9.4)
Requirements: The sample data consist of matched pairs. The samples are simple random samples. Either or both of these conditions are satisfied: The number of matched pairs of sample data is large (\( n > 30 \)) or the pairs of values have differences that are from a population having a distribution that is approximately normal.

Test Statistic: Compute the difference, and proceed with a one sample test as follows:

\[
t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}
\]

Degrees of freedom = \( n-1 \)

Hypothesis test for two Variances (Section 9.5)
Requirements: The two populations are independent of each other. The populations are each normally distributed.

Test Statistic: Compute the following ratio as follows:

\[
F = \frac{s_1^2}{s_2^2} \quad (\text{where } s_1^2 \text{ is the larger of the two sample variances})
\]

Numerator Degrees of freedom = \( n_1-1 \)
Denominator Degrees of freedom = \( n_2-1 \)