Multi-hop Qualitative Direction Estimation in Ad-Hoc Wireless Sensor Networks

GRADUATE PROJECT REPORT

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ABSTRACT

The project proposes a localization mechanism for supporting directional queries in ad hoc wireless sensor networks. Directional feature of wireless sensor networks is playing an important role in many research areas like ocean observation information analysis and other environment monitoring. In this work, four different algorithms have been applied to analyze the coarsely directional relationship between two different nodes in the wireless sensor networks. Each node in this wireless network has a fixed detecting radius, which means that each node can only retrieve its 1-hop neighborhood information. Those four developed algorithms will facilitate data retrieval between multiple-hop neighborhoods, and provide the directional relationship between any two nodes in this wireless sensor networks.
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1. BACKGROUND AND RATIONALE

1.1 Introduction

With the development of micro-electro-mechanical-system (MEMS) technology, wireless communication, and digital electronics, sensor nodes are becoming much cheaper and more functional, and starting consuming much less power than before. A wireless sensor network is composed of a large amount of sensor nodes, which are supposed to be densely deployed and implementing the function of sensing the neighbors, processing data and communicating information [Akyildiz 2001]. While wireless sensor networks have been applying widely in people’s life, such as vehicle control, disaster monitoring, and battlefield surveillance, more and more researcher and university’s scholars start focusing their work in this area, which brings wireless sensor networks a significant improvement. Additionally, widespread Internet technology helps computer users to monitor a phenomenon of interest remotely through wireless sensor networks as in Figure 1.1 [Mhatre and Rosenberg 2005].
1.2 Application of Wireless Sensor Networks

With their contributions, today wireless sensor networks have been used in a much wider range of applications, which could be roughly categorized into military, environment, health, home and other commercial areas.

1.2.1 Military Applications

Military requires a network to monitor the dynamic situation of the battlefield, which needs to be rapidly deployed, self-organized, and fault tolerated. The wireless sensor networks, which consist of large scale, low-cost, portable sensor nodes equipped in soldiers’ uniforms, are exactly qualified all the requirements, and the loss of any individual sensor node will not affect a military operation since the sensor nodes have been deployed densely in a large scale battlefield.

1.2.1.1 Monitoring friendly forces, equipment and ammunition

Wireless sensor nodes are able to be attached to every friendly troops and all the ammunitions, including vehicles, which helps commanders to have the real time status of their availability of the troops and ammunitions.

1.2.1.2 Battlefield surveillance and battle damage assessment

Wireless sensor networks are able to be deployed in critical terrains, routes, paths or straits rapidly, which helps military commander to watch opposing force closely in real time. Additionally, comparison of the number of sensor nodes before or after attacks will help the military to collect a coarsely battle damage assessment data.
1.2.2 Environmental Applications

Environmental applications of wireless sensor networks currently is one of the most important study aspects among all the other research directions regarding wireless sensor networks. Actually, today environmental applications of wireless sensor networks, such as forest fire detection, biological detection and Earth movement monitoring, play a very important role in people’s world [Agre and Clare 2000]. There are two typical examples are listed below.

1.2.2.1 Forest fire detection

A large scale of wireless sensor nodes will be deployed in a forest densely, randomly or maybe strategically. Once forest fire happens, this wireless sensor network will let the end users know the exact area of the fire, and it will help firefighters to put the fire down before it is becoming uncontrollable. Forest fire should not happen very frequently and the sensor nodes might be left unattended for month and even years, so an efficient and scavenging power source is becoming a solution, such as solar cells [Chandrakasan 1999]. This wireless sensor networks have much better performance in overcoming obstacles in the forest, like trees and rocks, in comparison to the traditional wired sensor networks.

1.2.2.2 Flood detection

A famous example of flood detection is the ALERT system [ALERT 2011], which is developed by the U.S. This ALERT system integrates all the information which has been collected from three different types of sensors, such as rainfall, water level and weather sensors.
There are still some other ongoing research projects regarding this topic in wireless sensor network filed, such as COUGAR Device Database Project at Cornell University [Bonnet 2000] and the DataSpace project at Rutgers [Imielinski and Goel 1999].

1.2.3 Health Application

Health application of wireless sensor networks in the hospital will help doctor to remotely monitor patient’s heart beating rate or blood pressure by the embedded or attached light-weight sensor nodes. This type of smart wireless sensor network will also help doctors to locate patients or maybe other doctors within the hospital in order to help doctors to make a faster response for some emergency situation [Bulusu 2001].

1.2.4 Home Application

Home application of wireless sensor networks is consist of many smart sensors which are embedded into all different appliances in the house, such as vacuum cleaner, micro-wave ovens, refrigerators, and VCRs [Petriu 2000]. These smart sensor nodes inside the device are able to interact with the other smart node and also with the external network through Internet or Satellite, which allows end users to manage home appliances locally or remotely much easier.

1.2.5 Other Commercial Application

Some of the commercial applications are monitoring material fatigue, managing inventory, monitoring product quality, robot control and guidance in automatic manufacturing environment, interactive toys and factory process control and automation and so on [Akyildiz 2001]. There is an example listed below.
1.2.5.1 Managing inventory control

Each item in the warehouse will be attached a smart sensor node, which contains item’s information in node’s memory. This wireless sensor network will help end users easily find out the exact location of the target item and also easily tally the number of items in the same category. This network is also feasible and easily extendible when adding new items to the inventory since all the users need to do is to attach the correct sensor nodes to the new items.

1.3 Constrain of Wireless Sensor Networks

Researchers create many advanced routing protocol and different topologies, which helps wireless sensor networks work more efficiently, however the severe limitation of sensor node’s battery energy has not been solved yet. Since the life cycle of sensor node’s battery decides the life cycle of the entire wireless sensor network, energy efficient protocols are required at all the layers of the protocol stack [Mhatre and Rosenberg 2005]. This limitation makes that sensor nodes have to be deployed densely in a typical wireless sensor network since node’s transmitting radius has to be adjusted to be short in order to consume less power. Moreover, wireless sensor nodes mostly have to be deployed randomly and densely since most of the applications are used in inaccessible terrains or disaster relief operations. Another interesting point of wireless sensor networks is that the topology of a sensor network changes very frequently. For example, if researchers want to forecast the path of hurricane in Gulf of Mexico, they have to deploy a very large amount of wireless sensor nodes into the ocean, which forms a dynamic topology because of the movement of the water. Figure 1.2 shows the most recent hurricane Ike projected path.
heading directly to the Texas Coast region while the eye of the hurricane close to the Jamaican islands.

![Projected path of Hurricane Ike](image)

Figure 1.2 Projected path of Hurricane Ike [Weather Channel 2011]

1.4 Reason for Retrieving Directional Relation

Many applications of wireless sensor networks, such as hurricane forecasting or earthquake forecasting, need location awareness to make the whole network work successfully and efficiently. However, due to the limitation mentioned above, each sensor node needs to adjust its power transmission to minimize the energy consumption. In other words, each node could only know the exact location information of their one-hop neighbors. GPS basically will solve this issue, and help the sensor node to find the exact location information of any other nodes in the network in outdoor environment. However, for large wireless sensor networks where nodes should be small, low power and cheap, putting a GPS chip in every single node is
too costly. Moreover, in most wireless sensor network applications, end users do not have to know the exact location relationship between two sensor nodes in the network, but a coarse one, such as “A is east of D”. This project proposes four different algorithms which allows to each node of the network to locate any other node coarsely in the network using only local information. This work is to show that in a wireless sensor networks where special hardware or GPS cannot be used for cost reason, there is a way to obtain coarse position of the nodes, even which are away from multiple hops [Heurtefeux and Valois 2008].
2. NARRATIVE

To achieve the goal of this paper, a wireless network model needs to be constructed first. The writer randomly selects 40 different points in a coordinate system, which forms a network as research object of this paper. A route will be generated between any two different points in the network, and later four different algorithms, which are the Frank’s method, the Hong’s method, the Heuristic Method and the Semi-Circular method, will be applied on this route in order to find out coarsely directional relationship between these two points.

2.1 Related works

Wireless sensor networks as a significant technology are attracting more and more researchers’ interests. Self-localization capability is one of the most highly desired characteristics of wireless sensor networks. Usually, sensor network localization algorithm estimate the locations of initially unknown sensor nodes by using the known positions of a few sensor nodes and inter-sensor measurements, such as distance. Sensors with known location information are called anchors and their locations can be obtained by using a global positioning system (GPS), or by installing anchors at points with known coordinates [Mao 2007].

In [Heurtefeux and Valois 2008], researchers categorize localization strategies into two types: fine and coarse localizations. The fine localization strategies determine nodes’ positions precisely by using their coordinates whereas the coarse localization strategies only offer estimated location information of unknown nodes in the network.
2.1.1 Fine localization strategies

GPS system is able to localize a node precisely. However, it is very costly to install a GPS receiver on each sensor node. Some papers introduce some methods to localize the nodes by using several anchors which are precisely located. In [Capkun 2001], researchers try to use ToA (Time of Arrival) to localize the node in the sensor networks. ToA allows the network system to compute the distance between two nodes by observing the time of propagation. However, this mechanism requires synchronization between different nodes. In [Ward 1997] and [Nawaz and Jha 2007], researchers create a new method TDoA (Time Difference of Arrival), which uses two signals of different natures, such as ultrasound and radio, to improve the result of ToA. In [Niculescu and Nath 2003] and [Akcan 2006], researcher use AoA (Angle of Arrival) to determine the direction of a radio wave propagation, and a combination method of both TDoA and AoA has been mentioned in [Capkun 2001] also. Actually, the preconditions of all above methods are: (1) energy consumption is not considerate, (2) we assume that each node is able to compute the time or angle of arrival easily.

2.1.2 Coarse localization strategies

Comparing to fine localization strategies, this one is aiming to find approximate coordinates of nodes in the wireless sensor networks. If a coarse location of the sensor node is acceptable in the network, there are several different mechanisms are available to be applied. The Active Badge System is mentioned in [Hopper 1993]. In this system, each node is labeled and transmits a periodic hello packet every 10 seconds with a unique infra red signal which is received by dedicated sensors placed at fixed positions within a building, and relayed to the location manager. Location Estimation Algorithm mentioned in [Hu and Evans 2004], provides a
probabilistic distribution of the possible node locations. By combining the previous location information and new observations from anchor nodes, impossible location will be parsed out. Finally, the virtual coordinates is mentioned in [Cao and Abdelzaher 2006], which constructs a coordinate system by computing the distance between each node and anchors in number of hops.

In [Heurtefeux and Valois 2008], authors also offer a new algorithm which is to determine coarsely the location of the neighbors of a selected node by using only local information, and those local information is collected from hello packets which are exchanged between 1-hop neighbors. Additionally, Dr. Longzhuang Li, who is professor in Texas A&M University – Corpus Christi, also put efforts on finding out the directional relations between indirectly connected observing stations around the Gulf of Mexico area. In [Li 2010], a semi-circular algorithm is used to determine the directional relation between different stations, which could be considered as prototype of this paper.

2.2 Topology

The selected 40 different points are listed in Table 2.1, and their neighborhood topology has been shown in Figure 2.1 also. We assuming the transmitting radius of any individual node are same which is defined as 5, and any node only can have the exact location information of its neighbors which are within the transmitting radius. For example, in Figure 2.1, the only neighbor of R is S since the distance between R and T is greater than the transmitting radius. Thus, we know that S is southwest of R, but we do not know the directional relation between R and T since the distance between these two points is out of range. Thus, if the users want to know the directional relationship between R and T, they have to use a middle connecting point S by
applying four different algorithms introduced later. Moreover, in this wireless sensor network model, the distances between nodes are classified in very close, close, medium, far and very far.

Any link in Figure 2.1 smaller than transmitting radius, which is 5, will be defined as very close; any link greater than 5, but smaller than 10 is defined as close; any link greater than 10, but smaller than 15 is defined as medium; any link greater than 15, but smaller than 20 is defined as far; any link greater than 20 is defined as very far.

Table 2.1 Table showing the coordinates of the selected points

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| X | 11 | 0  | -1 | -5 | -4 | -6 | -5 | -6 | -5 | -2 | -5 | -5 | -7 | -10 | -8 | -11 | -12 |   |
| Y | 1  | 0  | -1 | -5 | -4 | -6 | -5 | -6 | -5 | -2 | -5 | -7 | -10 | -8 | -11 | -12 |   |

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Figure 2.1 Neighborhood graph of the research model
2.3 Directional relation

Qualitative direction is a function between two points in the plane that maps onto a symbolic direction. The \( n \) different symbols available for describing the directions are given as a set \( C_n \). The value of \( n \) depends on the specific system of direction used, e.g., \( C_4 = \{ \text{North, East, South, West} \} \) or more extensively \( C_8 = \{ \text{North, Northeast, East, Southeast, South, Southwest, West, Northwest} \} \) (see Figure 2.2). In the paper, we employ the cone-shaped direction system \( C_8 \). For instance, the direction \textit{Northeast} refers to a cone-shaped section with the angular degree \([22.5^\circ, 67.5^\circ]\). This type of direction system results in the feature that the allowance area for any
given direction increases with distance [Frank 2006]. In this paper, we use the angular values, 
\[ \pi/2, \pi/4, 0, 3\pi/4, 5\pi/4, \pi, \text{ and } 3\pi/4 \] to represent eight directions in \( C_8 \), respectively.

In the qualitative directional reasoning, we should be able to reason directional information of indirectly connected points, or the composition of two directional relations to derive a new directional relation. A typical composition is like: given \( B \) is east of \( A \) (\( C_{AB} = \text{East} \)) with the angular value \( \alpha \) and \( C \) north of \( B \) (\( C_{BC} = \text{North} \)) with the angular value \( \beta \), what is the direction \( C_{AC} \) between \( A \) and \( C \) (see Figures 2.3(b), (c), (d))? The all-answer composition results of the eight cone-shaped cardinal direction relation are shown in Table 2.2 [Frank 2006] [Ligozat 1998]. For 64 compositions in Table 2.2, only 8 can be inferred exactly with one answer, 8 (represented by *) may be any of the eight directions, and the remaining 48 generate two possible answers, three possible answers, or four possible answers. The reason is that the mere directional relationships do not provide enough information to infer the result of a composition. For example, by checking the intersection of the fourth row and the second column, \( C_{AC} \) may be \textit{East} (see Figure 2.3(b)), \textit{Northeast} (see Figure 2.3(c)), or \textit{North} (see Figure 2.3(d)) depending on not only directions but also distances between \( A \) and \( B \) (\( L_{AB} \)) and between \( B \) and \( C \) (\( L_{BC} \)). In our application domain where the coordinates of the points are not available, we need to provide a unique and precise answer for each user’s spatial query instead of two, three, or four possible answers. Next, we present four methods to reason a unique composition answer. In the following discussion, we number each direction clockwise starting from \textit{North} as 1 and ending with \textit{Northwest} as 8, e.g., the direction gap between \textit{North} and \textit{East} is 2.
Table 2.2 The all-answer composition for eight cone-shaped cardinal direction relations

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Figure 2.3 The eight direction system and a composition example when $|\beta - \alpha| = \pi/2$.

2.3.1 The Frank’s method

Frank [Frank 1992][Frank 2006] derived the single answer table (see Table 2.3) based on some directional properties and two averaging rules. One of the most useful properties in constructing Table 2.3 is that the composition of two line segments with the same direction remains the same direction. In addition, the average rules state that the composition of two
directions is: (1) if the direction difference is 2, the result is the one in the middle of two directions; (2) if the direction difference is 1, one of the two directions is selected randomly. For example, S combined with E leads to SE, and S combined with SE results in SE or S.

As a result, only 8 cases can be inferred precisely according to the directional property, 32 give approximate results based on the two average rules, and other 24 are undetermined (represented as U). In the rest of the paper, this method is referred to as the Frank92 method.

Table 2.3 The single-answer composition results from [Frank 1992][Frank 2006]

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2.3.2 The Hong’s method

In the Hong’s method [Hong 1995], the directional reasoning is derived from the composition of two sets of information: cone-shaped qualitative directions and qualitative distances. To be more specific, Hong presented three reasoning models, the all-answer model, the likely-answer model, and the single-answer model. The all-answer model finds all the possible answers, the likely-answer model removes results that have a low probability, and the single-answer model chooses the single composition with the highest probability.

The three models are evaluated and tested in [Hong 1995] based on (1) eight qualitative directions, (2) four qualitative distances, and (3) thirteen length ratios. The eight qualitative
directions are the same as the cone-shaped direction system $C_S$ mentioned above, the four qualitative distances are near, medium, far, very far. The length ratio is its ratio to the neighboring distances. The thirteen length ratios are 1 to 10, 20, 50, and 100. In the simulation, 100 samples are uniformly distributed in the individual distance and direction sector. As a result, the total number of possible combination between two sectors is 10,000. We are especially interested in the single-answer model, which is referred as the Hong95 method.

It is discovered by simulation that all compositions are robust if the length ratio $\rho = \frac{L_{BC}}{L_{AB}}$ is greater than 2 or less than 0.5. Here robust means that the composition results will be the same no matter how the length ratio changes as long as $\rho > 2$ or $\rho < 0.5$. For example, the composition result $C_{AC}$ of $C_{AB}$ and $C_{BC}$ will always be Southwest if $\rho > 2$ and $C_{BC}=$Southwest (see the seventh column in Table 2.4). Table 2.5 shows the composition results if $\rho < 0.5$, and table 2.6 presents the composition results when $\rho = 1$. In table 2.6, 8 compositions are undetermined (represented by $U$).

**Table 2.4** The composition results when $\rho > 2$ in [Hong 1995]

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Table 2.5 The composition results when $\rho < 0.5$ in [Hong 1995]

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Table 2.6 The composition results when $\rho = 1$ in [Hong 1995]

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2.3.3 The Heuristic method

The heuristic method is developed from a probabilistic method proposed by [Dehak, Bloch and Maitre 2005] to uniquely determine the angular relationship \( r \) between two points, \( A \) and \( C \), giving the angular value \( \alpha \) from \( A \) to \( B \) and the angular value \( \beta \) from \( B \) to \( C \) [Li, Liu, Nalluri and Jin 2009].

Under the assumption of uniform distribution for all points in a circular region and no knowledge of the point coordination information (longitude and latitude), \( r \) is calculated as:

\[
r = \begin{cases} 
\frac{\alpha + \beta}{2} & \text{if } |\beta - \alpha| \in \left[ 2k\pi, 2k\pi + \frac{\pi}{2} \right] \\
\pi + \frac{\alpha + \beta}{2} & \text{if } |\beta - \alpha| \in \left[ 2k\pi + \frac{3\pi}{2}, (2k+2)\pi \right] \\
\frac{\varphi(\alpha, \beta)}{\sigma(\alpha, \beta)} & \text{if } |\beta - \alpha| \in \left( 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{3\pi}{2} \right), \quad \alpha < \beta, \quad |\beta - \alpha| \neq \pi \\
\frac{\pi + \varphi(\alpha, \beta)}{\sigma(\alpha, \beta)} & \text{if } |\beta - \alpha| \in \left( 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{3\pi}{2} \right), \quad \alpha > \beta, \quad |\beta - \alpha| \neq \pi \\
\alpha + \frac{6\pi}{7} & \text{if } \beta = \alpha + \pi, \quad \text{or} \quad \alpha - \frac{6\pi}{7} & \text{if } \alpha = \beta + \pi
\end{cases}
\]

with

\[
\varphi(\alpha, \beta) = -4((2\beta - \alpha - \pi)^2 - \beta^2 + 1) \cos(\alpha - \beta) + 5 \cos(3\alpha - 3\beta) - \cos(5\alpha - 5\beta)
- (12\alpha - 20\beta + 8\pi) \sin(\alpha - \beta) + (8\alpha - 10\beta + 5\pi) \sin(3\alpha - 3\beta)
- (2\beta - \pi) \sin(5\alpha - 5\beta),
\]

and

\[
\sigma(\alpha, \beta) = 8(\alpha - \beta + \pi) \cos(\alpha - \beta) + 8 \sin(\alpha - \beta) - 2 \sin(3\alpha - 3\beta) - 2 \sin(5\alpha - \beta)
\]

Equation (1) works well if two distances (\( AB \) and \( BC \)) are similar.
In this application domain, we have the knowledge of cardinal directions, such as north or southwest, between two points, but the real angular information between two points is unknown. So we have to employ the average angular degree of a cone-shaped sector to represent the angle of a direction. For example, the direction northeast is between $\pi/8$ and $3\pi/8$ and is represented by the average angle $\pi/4$. In this case, the values of $\alpha$ and $\beta$ can only be $0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2$, or $7\pi/4$ for eight directions, respectively. As a result, Equation (1) only returns two types of $r$ values when $|\beta - \alpha|$ is in the first or fourth quadrant: $r$ is right on the delimiter of two cone-shaped sectors when $|\beta - \alpha| = \pi/4$ or $7\pi/4$, or $r$ is the average angle of a cone-shaped sector when $|\beta - \alpha| = 0, \pi/2$, or $3\pi/2$. For instance, $\alpha = 0, \beta = \pi/4$, then $r = (\alpha + \beta)/2 = \pi/8$, which is the degree that separates east and northeast. In the above example, to determine a direction for $AC$, we may use some heuristics or randomly pick one if the two distances, $AB$ and $BC$, are the same.

On the other hand, the two distances can vary widely, for example, $AB$ is medium and $BC$ is far in Figure 2.3 (d). If we do not take the distance into consideration, the result of Equation (1) may drift away from the real angle between $A$ and $C$. To handle the above disparate distance problem, we propose heuristics to improve the performance of Equation (1) according to the $|\beta - \alpha|$ value and length of $AB$ and $BC$. Next, we discuss the heuristics proposed for various scenarios.

**Heuristics 1:** When $|\beta - \alpha| = \pi/2$ or $3\pi/2$, $AC$ is in the same direction with the longer one of $AB$ and $BC$, or $AC$ is in the same direction with the middle section between $AB$ direction and $BC$ direction if $AB$ and $BC$ are of same length. For example, in Figure 5, $\alpha = 0, \beta = \pi/2$, and $r = \pi/4$ according to Equation (1). Suppose the distance of $AB$ is medium, $C$ is east of $A$ if $BC$ is close
(see Figure 2.3 (b)), C is northeast of A if $BC$ is medium (see Figure 2.3 (c)), or C is north of A if $BC$ is far (see Figure 2.3 (d)).

**Heuristics 2:** When $|\beta - \alpha| = \pi/4$ or $7\pi/4$, $AC$ is in the same direction with the longer one of $AB$ and $BC$, or $AC$ is in the same direction with $BC$ if $AB$ and $BC$ are of same distance. For example, $B$ is east of $A$ and $C$ is northeast of $B$, then $\alpha = 0$, $\beta = \pi/4$, and $r = \pi/8$ according to Equation (1). Suppose the distance of $AB$ is medium, $C$ is east of $A$ if $BC$ is close (see Figure 2.4 (a)), or $C$ is northeast of $A$ if $BC$ is medium or far (see Figure 2.4 (b) and 2.4 (c)).

![Figure 2.4 Composition results when $|\beta - \alpha| = \pi/4$](image)

**Heuristics 3:** When $|\beta - \alpha| = 3\pi/4$ or $5\pi/4$, the direction of $AC$ is determined by the result of Equation (1) if $AB$ and $BC$ are of same length. Otherwise, $r$ is increased/decreased by $\pi/8$ if $AB$ is shorter than $BC$, or $r$ is increased/decreased by $\pi/2$ if $AB$ is longer than $BC$. The reason we do this is because usually the value $r$ falls in the third or fourth direction from $AB$ direction. For example, in Figure 2.5, $B$ is east of $A$ and $C$ is northwest of $B$, then $\alpha = 0$, $\beta = 3\pi/4$, and $r \approx 1.95$, which is about $112^\circ$ and north of $A$ according to Equation (1). Suppose the distance of $AB$ is medium, in Figure 2.5 (a) $C$ is east of $A$ because $BC$ is close and $r$ is decreased by $\pi/2$ to 0.38 (about $22^\circ$), while in Figure 2.5 (d) $C$ is northwest of $A$ since $BC$ is far and $r$ is increased by $\pi/8$.\n
to 2.34 (about 134°). Figures 2.5 (b) and 2.5 (c) show two possible locations of C when AB and BC are of same length.

![Diagrams](image1)

Figure 2.5 Composition results when $|\beta - \alpha| = \frac{3\pi}{4}$.

Based on the three heuristics, the composition results are shown in tables 7, 8, and 9 when $L_{AB} = L_{BC}$, $L_{AB} < L_{BC}$, and $L_{AB} > L_{BC}$, respectively. Here we are comparing two qualitative distances, which can be very close, close, medium, far, or very far.

**Table 2.7** The composition results when $L_{AB} = L_{BC}$ in [Li, Liu, Nalluri and Jin 2009]

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**Table 2.8** The composition results when $L_{AB} < L_{BC}$ in [Li, Liu, Nalluri and Jin 2009]

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Table 2.9 The composition results when $L_{AB} > L_{BC}$ in [Li, Liu, Nalluri and Jin 2009]

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2.3.4 The Semi-Circular Method

The semi-circular normal distribution is used to model a circular random variable that does not exceed half circle. This feature makes it naturally fit our direction reasoning application, where the information in some certain direction is sought. The method is referred as the semi-circular method later on.

The semicircular normal distribution and the normal distribution are equivalent at the limit for small angles but the former is more appropriate for circular data. The semicircular normal (SCN) distribution is obtained by projecting a normal distribution over a semi-circular segment [Guardiola 2006]. Let $x$ have a normal distribution with mean zero and variance $\sigma^2$. For any positive real number $\rho$, the angle $\theta$ is defined by

$$\theta = g(x) = \arctan\left(\frac{x}{\rho}\right)$$  \hspace{1cm} (2)

Hence $x = g^{-1} = \rho \tan(\theta)$. It follows that $\frac{dx}{d\theta} = \rho \sec^2(\theta)$ and the density of the angle $\theta$ is
\[ f(\theta) = f(g^{-1}(\theta)) \frac{dx}{d\theta} = \frac{\rho}{\sqrt{2}\pi\sigma} \sec^2(\theta) \exp \left[ -\frac{\rho^2 \tan^2(\theta)}{2\sigma^2} \right], \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \ \sigma, \ \rho \in \mathbb{R}^+. \] (3)

Let \( \varphi = \sigma/r \). Then, allowing for negative angles, we have

\[ f(\theta) = \frac{1}{\sqrt{2}\pi\varphi} \sec^2(\theta) \exp \left[ -\frac{\tan^2(\theta)}{2\varphi^2} \right], \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \ \varphi \in \mathbb{R}^+. \] (4)

We refer to (4) as the semicircular normal (SCN) density and write \( \theta \sim \text{SCN}(0, \varphi) \). Notice that (4) is not defined at \( \theta = \pm \pi/2 \) as \( \cos(\pi/2) = 0 \) and therefore \( \sec^2(\pi/2) \) is not defined. However, \( \lim_{\theta \to \pm \pi/2} f(\theta) = 0 \). More generally, the parameter \( \mu \) is introduced as the location parameter for the SCN relative to the horizontal axis. The density can be written as

\[ f(\theta|\mu) = \frac{1}{\sqrt{2}\pi\varphi} \left[ \sec(\theta-\mu) \right]^2 \exp \left[ -\frac{[\tan(\theta-\mu)]^2}{2\varphi^2} \right], \quad -\frac{\pi}{2} + \mu < \theta < \frac{\pi}{2} + \mu, \ \varphi \in \mathbb{R}^+. \] (5)

It is straightforward to show that the cumulative distribution for \( \theta \) is

\[ F(\alpha) = \Phi \left[ \frac{1}{\varphi} \tan(\alpha - \mu) \right], \] (6)

where \( \Phi \) is the standard normal cumulative distribution function (CDF). Equivalently, \( F(\alpha) = (1/2)\text{erf} \left[ \tan(\alpha - \mu) / \sqrt{2}\varphi \right] \), where \( \text{erf} \) is the error function defined by the \( \text{erf}(x) = (2/\sqrt{\pi}) \int e^{-x^2} \, dx \) function. The function \( \text{erf}(x) \) differs from the CDF of a normal distribution only by a multiplicative constant. See for example in [Spiegel 1968]. The standard deviation expressed in terms of the parameter \( \sigma \) of the normal distribution is shown in Table 2.10 and it is equivalent to the parameter \( \varphi \) of the semicircular normal distribution after an appropriate transformation. This transformation is done with the purpose of increasing readability.

According to the above described SCN distribution, the approximated composition angle degree \( \theta \) and corresponding standard deviation can be obtained by considering the length ratio \( r \)
as a random variable that follows a continuous uniform distribution with a 20% range of variability (20% is chosen to allow certain flexibility for qualitative distance). The described random variable $r$ models the length ratio of the two line segments involved. For example, two line segments $AB$ and $BC$ have the same qualitative distance, $L_{BC}=L_{AB}=\text{close}$, and the length ratio is modeled using a continuous uniform distribution with a range of plus and minus 20%, then the length ratio $\rho$ of $L_{BC}/L_{AB}$ is uniformly distributed within the range of [0.8, 1.2], the composition angle degrees $\theta$ and standard deviation expressed in degrees are shown in Table 2.10 by computing the mean and standard deviation of 1000 iterations. For a better understanding of Table 2.10, let us look at the cell in the second column, the label $N$ means that station B is to the North of station A, similarly the label $N$ in the first row says that station C is to the North of station B. The semi-circular method computes the mean angle and its equivalent standard deviation from station A to station C as 90.1° and 8.8°, respectively. The angles expressed in degrees in Table 2.10 are transformed to their corresponding cone-shaped cardinal directions as shown in Table 2.11.

**Table 2.10** The composition angle and standard deviation when $\rho$ is in [0.8, 1.2]

<table>
<thead>
<tr>
<th>ANGLE</th>
<th>N</th>
<th>NE</th>
<th>E</th>
<th>SE</th>
<th>S</th>
<th>SW</th>
<th>W</th>
<th>NW</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANGLE</td>
<td>N</td>
<td>90.1</td>
<td>67.8</td>
<td>45.4</td>
<td>22.7</td>
<td>1.9</td>
<td>156.9</td>
<td>134.8</td>
</tr>
<tr>
<td>STD</td>
<td>8.8</td>
<td>8.9</td>
<td>9.5</td>
<td>17.5</td>
<td>109.6</td>
<td>17.2</td>
<td>9.5</td>
<td>8.9</td>
</tr>
<tr>
<td>ANGLE</td>
<td>NE</td>
<td>67.5</td>
<td>44.2</td>
<td>22.8</td>
<td>0.4</td>
<td>338.5</td>
<td>22.6</td>
<td>112.3</td>
</tr>
<tr>
<td>STD</td>
<td>8.9</td>
<td>8.8</td>
<td>8.9</td>
<td>9.5</td>
<td>17.6</td>
<td>99.6</td>
<td>18.5</td>
<td>9.5</td>
</tr>
<tr>
<td>ANGLE</td>
<td>E</td>
<td>44.8</td>
<td>22.6</td>
<td>0.1</td>
<td>337.8</td>
<td>315.4</td>
<td>293.6</td>
<td>182.3</td>
</tr>
<tr>
<td>STD</td>
<td>9.5</td>
<td>8.9</td>
<td>8.8</td>
<td>8.9</td>
<td>9.5</td>
<td>18.6</td>
<td>100.0</td>
<td>19.1</td>
</tr>
</tbody>
</table>
The tables for other length ratios $\rho$ and corresponding standard deviations $\sigma$ are calculated in a similar manner. Table 2.12 shows the results where $AB$ is shorter than $BC$ and the qualitative distance gap between $AB$ and $BC$ is 1, e.g., $L_{AB}=\text{close}$ and $L_{BC}=\text{medium}$. Table 2.13 displays the results where $AB$ is longer than $BC$ with the qualitative distance gap of 1, e.g., $L_{AB}=\text{medium}$ and $L_{BC}=\text{close}$. In this paper, the qualitative distance gap of 1 means that the ratio between the quantitative distance $Q_{BC}$ and $Q_{AB}$, $Q_{BC}/Q_{AB}$, is about 2 or 3. We use the average of two composition angles to represent the case where the $Q_{BC}$ is about two or three times of $Q_{AB}$.

For example, for a range variability of plus and minus 20%, direction $C_{AB}=\text{North}$ and $C_{BC}=\text{Southeast}$, angle $\theta=342.39^\circ$ if $Q_{BC}/Q_{AB}=2$ and $\theta=331.55^\circ$ if $Q_{BC}/Q_{AB}=3$, then the average angle of two $\theta$ values is $336.97^\circ$ and the corresponding direction is $\text{Southeast}$ as shown in Table 2.12. The results in Table 2.12 are almost the same as those in Table 2.4 of the Hong95 method except one composition, where the combination of NE and SE is E. Similarly, the composition
results are shown in Table 2.13 when $Q_{BC}$ is about one half or one third of $Q_{AB}$. Table 2.13 and Table 2.5 only differ in one value, where the composition of SW and N is W. The composition results of the semi-circular method are the exact same as those of the Hong95 method when the length ratio is greater than 3 or less than one third. The above comparisons illustrate that the major difference between the semi-circular method and the Hong95 method lies in the case where $AB$ and $BC$ have the similar qualitative distance.

**Table 2.11** The composition direction when $\rho$ is in $[0.8, 1.2]$

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>NE</th>
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</table>

**Table 2.12** The composition when $\rho$ is in $[1.6, 3.6]$

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<thead>
<tr>
<th></th>
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</table>
Table 2.13 The composition when $\rho$ is in [0.2666, 0.6]

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3. SYSTEM DESIGN

The project is implemented using Microsoft visual studio 2008 C++. In this chapter, the author introduces development environment of the project, then analysis, design, and implementation phases keeping software engineering point of view.

3.1 Data Retrieval Phase

The 40 selected points’ coordinates are saved in a text file, which is named “plot.txt” and saved in the path of C:\Users\UserName\Documents\Visual Studio 2008\Projects\ProjectName\ProjectName\plot.txt. In the data path shown above, user needs to use its own user’s name and project’s name to fill up the bold section.
Figure 3.1 Snapshot of data retrieval (a)
3.2 Design Phase

Firstly, the designed system has to be able to generate any two different points among those 40 points in plane which we selected earlier, and those two points have to reach the requirements: (1) two different points, (2) the distance between two points has to be greater than sensor node’s transmission radius otherwise the nodes will know each other’s location and this work will be useless. Moreover, we have to define an array which holds all the two points combinations picked earlier. Once designed system generates a new combination, system will use it to compare with all combinations stored in the array earlier and (1) if there is a duplicated combination, system will generate a new combination and repeat the steps listed above, (2) otherwise, save the selected combination into the array, and start working on the research.
between those two points. Additionally, the first point generated in the combination will be the starting points, and the other one is the finishing point. Actually, if we reverse the starting point and finishing point, there might be another different routing path generated by the operation. However, in this paper, duplication removal function mentioned above will help system remove all the same combination, thus, in other words, there will not be any combination selected twice regardless the sequence within the combination. Then, the routing protocol defined in this paper is that (1) starting points will collect all the coordinates within its transmission radius and save them into an array, (2) the new starting point of next hop is the one nearest to the finishing point among all selected coordinates in the first step, (3) keep repeating first two steps until find the finishing point (see figure 3.3).
After routing path is constructed, we translate four different algorithms into different

Table 2 and try to find out directional relation between nodes. Except for the

Frank’s method, all the other three algorithms are closely related to the distance between nodes.

In other words, there is a key parameter in those algorithms which is \( \rho = \frac{L_{BC}}{L_{AB}} \). However, since

the whole network is restricted by transmission radius, each sensor node only can find their very
close neighbors. In this paper, the quantitative distance between AB and BC will be described as

\( Q_{AB} \) and \( Q_{BC} \), and \( \rho = \frac{Q_{BC}}{Q_{AB}} \). Thus, the first step of all the routes in the network is computed

with the parameter of \( \rho = \frac{Q_{BC}}{Q_{AB}} = 1 \), since the distances of both AB and BC are very close

(see Figure 3.4). After the first step, in Figure 3.4, AC is becoming the new AB and CD is

becoming the new BC, and \( \rho \) will be less than 1 always since \( \rho = \frac{Q_{CD}}{Q_{AC}} \) and distance

between B and C is very close since C is still in the transmission radius of B (see CD in Figure

3.4), and distance between A and C is any other category of quantitative distance greater than

very close since C is never in the transmission radius of A.
3.3 Implementation Phase

Corresponding to Figure 3.3, the blow snapshots will demonstrate the procedure of the
code implementation. Figure 3.5 shows the randomly selected two points among 40 preselected

The randomly selected two points are:
[-7, -6]
[14, 14]
coordinates.

Figure 3.5 Randomly selected two points

Figure 3.6 shows all the neighbors of node A [-7, -6] (see Figure 3.3) within the sensor node’s transmitting radius. And Figure 3.7 indicates that the next selected point of the routing path is B [-4, -4] (see Figure 3.3 also) which is within A’s transmitting radius and nearest to the destination point H [14, 14]. Figure 3.8 shows that a typical routing path has been constructed by keeping repeating the above steps. Eventually, Figure 3.9 shows that the final outcomes of directional estimation in this studying prototype network by using four different algorithms.

Figure 3.6 All the neighbors of A [-7, -6]
Figure 3.7 The selected next point is B [-4, -4]

Figure 3.8 The final routing path is selected
4. EVALUATION AND RESULTS

Evaluation:

Testing is important for any project. Testing is all about what is specified and what is delivered. It verifies the performance and implementation of the project. In this project, we embed many debug codes between statements, and those debug codes can make sure the output value from the project is the same as we expected when we designed it. Firstly, we need to make sure all the coordinates read from the text file should be completely correct. Secondly, we have to make sure the routing path in this network is correct. Finally, we have to make sure that all algorithms have been coded properly and all the final outcomes are reasonable.
Results:

In the experiments, the correctness of the above four methods, the Frank92 method, the Hong95 method, the heuristic method, and the proposed semi-circular method, is tested on 600 compositions from the designed wireless sensor network (see Figure 2.1). For cases where the composition results are $U$ (undetermined) in Tables 2.3 and 2.6, the composition results are randomly selected from 8 directions. The five qualitative distances, close, very close, medium, far, and very far are defined in our distance system corresponding to Hong95 method.

The testing results on 600 compositions are shown in Table 4.1. Correct ratio (CR) is calculated as $(\text{correct inference (CI)}) / (\text{total compositions})$. For example, CR of the semi-
circular method is 362/600=60.33%. Gap score (GS) is the directional difference between the inferred direction and the real direction. For example, the GS is 1 if the inferred direction is east and the real direction is northeast. In Table 4.1, the Hong95 and Semi-circular methods provide similar performance in terms of the CR, which are 59.33% and 60.33%, respectively. The Heuristic method has the CR of 57.17%, which is lower than that of the Hong95 and Semi-circular methods but is still significantly better than the Frank92 method. The results are understandable because the Frank92’s results are based on directional information only. The Semi-circular method generates the smallest GS. To be more specific, the Semi-circular method achieves the GS of 310, the Hong95 method 312, the Heuristic method 335, and the Frank92 method 334, respectively.

Additionally, sometimes there is more than one path to reach from one node to another in the composition operation, however, due to the restriction of the routing protocol in this project (see Figure 3.3), those situations are not considerate.

**Table 4.1** Comparison of composition results of four methods

<table>
<thead>
<tr>
<th></th>
<th>Frank 92</th>
<th>Hong 95</th>
<th>Semi-Circular</th>
<th>Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct Inference</td>
<td>339</td>
<td>356</td>
<td>362</td>
<td>343</td>
</tr>
<tr>
<td>Gap Score</td>
<td>334</td>
<td>312</td>
<td>310</td>
<td>335</td>
</tr>
<tr>
<td>Correct Ratio</td>
<td>56.5%</td>
<td>59.33%</td>
<td>60.33%</td>
<td>57.17%</td>
</tr>
</tbody>
</table>
5. Additional Experiment

Above research is based on a basic assumption which we assume that all the nodes in the wireless sensor network have identical transmitting radius. If each sensor node in the network has different transmitting radius, what will be the result of the research? In the following experiments, we focus on the correct ratio of the routes which are constructed by three, four and five randomly selected nodes respectively. The correctness of the above four methods, the
Frank92 method, the Hong95 method, the heuristic method, and the proposed semi-circular method, is tested on 200 compositions of each experiment from the designed wireless sensor network (see Figure 2.1). Additionally, all the statistics in Table 5.1, 5.2, and 5.3 are the average results out of ten times experiments.

Table 5.1 Comparison of composition results constructed by three randomly selected nodes

<table>
<thead>
<tr>
<th>Routes with four nodes</th>
<th>Frank 92</th>
<th>Hong 95</th>
<th>Semi-Circular</th>
<th>Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct Inference</td>
<td>65</td>
<td>117</td>
<td>114</td>
<td>93</td>
</tr>
<tr>
<td>Correct Inference</td>
<td>93</td>
<td>125</td>
<td>131</td>
<td>115</td>
</tr>
<tr>
<td>Gap Score</td>
<td>261</td>
<td>105</td>
<td>100</td>
<td>168</td>
</tr>
<tr>
<td>Gap Score</td>
<td>194</td>
<td>86</td>
<td>79</td>
<td>117</td>
</tr>
<tr>
<td>Correct Ratio</td>
<td>32.5%</td>
<td>58.5%</td>
<td>57%</td>
<td>46.5%</td>
</tr>
<tr>
<td>Correct Ratio</td>
<td>46.5%</td>
<td>62.5%</td>
<td>65.5%</td>
<td>57.5%</td>
</tr>
</tbody>
</table>

Table 5.2 Comparison of composition results constructed by four randomly selected nodes

<table>
<thead>
<tr>
<th>Routes with five nodes</th>
<th>Frank 92</th>
<th>Hong 95</th>
<th>Semi-Circular</th>
<th>Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct Inference</td>
<td>55</td>
<td>112</td>
<td>91</td>
<td>80</td>
</tr>
<tr>
<td>Gap Score</td>
<td>288</td>
<td>117</td>
<td>125</td>
<td>193</td>
</tr>
<tr>
<td>Correct Ratio</td>
<td>27.5%</td>
<td>56%</td>
<td>45.5%</td>
<td>40%</td>
</tr>
</tbody>
</table>
Results:

In Table 5.1, each composition is constructed by three randomly selected sensor nodes, and Hong’s method and Semi-Circular method provide very close performance. The Correct ratio (CR) of Hong’s method and Semi-Circular method are 62.5% and 65.5% respectively, and Semi-Circular method has very faint better performance in comparison to Hong’s method. The Heuristic method has a little bit weaker performance than above two, but it still maintains much better correct ratio than Frank’s method. In Table 5.2, Hong’s method and Semi-Circular method are still maintaining their dominant positions on the correct ratios, however, contrarily, this time Hong’s method has very faint better performance in comparison to Semi-Circular method. Additionally, the correct ratio of Frank’s method drops much faster than all the other three significantly. In Table 5.3, Hong’s method has the best correct ratio, and Semi-Circular and Heuristic methods have closer results which are still having much better performance than Frank’s method. Moreover, if we look at above three tables by columns, it is easy to tell that while more sensor nodes are involving to the routes, the Correct Ratios of all four algorithms are becoming lower and Gap Score are increasing generally. Eventually, Hong’s method’s correct ratio drops much slower than all the other three, and Hong’s and Semi-Circular method maintains much better performance on the Gap Score, which means the correct ratios of Hong’s and Semi-Circular method could be easier tolerated by some additional algorithms than the other two.
6. FUTURE WORK

Future research includes: (1) above four algorithms need to be applied in a more complicated wireless sensor network, which include isolated nodes and self-tolerated function to find connectivity in the wireless sensor networks. Moreover, the larger size network will bring researchers more combinations of the nodes. In this work, our study has been limited by 780
combinations among 40 different points; (2) as we mentioned in last chapter, sometimes there is more than one path to reach from one node to another in the composition operation, so the final correct ratio can be adjusted higher to some extent; (3) we need to compare the performance of above four composition methods on points of different distribution; (4) in this work, quantitative distance has been generated by real coordinates of selected points, however, most of time the real coordinate of the node is not available. Today, some scholars are working on finding out quantitative distance in the wireless sensor networks by using network’s topology and connectivity to close neighbors [Heurtefeux and Valois 2008], and we could combine those contributions to our research, which could make our project be applied in much wider fields.

7. CONCLUSION

Combining the qualitative distance and directional information, the proposed statistic-based semi-circular method as well as the existing probabilistic-based heuristic method and Hong’s method obtain higher accuracy than Frank’s method which only utilizes directional
information. Moreover, the Semi-circular method and Hong’s method achieve very close best results among the four directional reasoning methods since the composition results of the semi-circular method when $\rho < 1/3$ are the exact same as those of the Hong95 method when $\rho < 0.5$ (see table 2.5). Additionally, when $1/3 < \rho < 0.5$, Table 2.13 and Table 2.5 only differ in one value, where the composition of SW and N is W. Thus, for $\rho < 0.5$, the Semi-circular and Hong95 almost have the same composition results. The above comparisons illustrate that the major difference between the Semi-circular method and the Hong95 method lies in the case where $AB$ and $BC$ have the similar qualitative distance which is basically very first step of constructing the routing path (see figure 3.4). In conclusion, in a large size wireless sensor network, the directional relation between two nodes, which are multiple hops away from each other, is more decided by the exact known directions of first couple pieces of the routing path.

8. BIBLIOGRAPHY


[Li, Liu, Nalluri and Jin 2009] Li L., Liu Y., Nalluri A. and Jin C., Qualitative spatial representation and reasoning for data integration of ocean observational systems. In The Joint International Conferences on Asia-Pacific Web Conference (APWeb) and Web-Age Information Management (WAIM).


APPENDIX – A
1. Node- A node is a critical element of any computer network. It can be defined as a point in a network at which lines intersect or branch, a device attached to a network, or a terminal or other point in a computer network where messages can be transmitted, received or forwarded.

2. Topology - It is the study of the arrangement or mapping of the elements (links, nodes, etc.) of a network, especially the physical (real) and logical (virtual) interconnections between nodes.

3. R-Project: R is a language and environment for statistical computing and graphics. R provides a wide variety of statistical (linear and nonlinear modeling, classical statistical tests, time-series analysis, classification, clustering, etc) and graphical techniques, and is highly extensible.

APPENDIX – B
Statistical Simulation code:

```r
set.seed(10)
install.packages('base')

n <- 1000
sd <- 12
sd <- sd*pi/180

mu1 <- -45
mu1 <- mu1*pi/180
theta1 <- rnorm(n, mu1, sd)
theta1

mu2 <- -45
mu2 <- mu2*pi/180
theta2 <- rnorm(n, mu2, sd)
theta2

k <- runif(n, .9, 1.1)
k

rx <- cos(theta1) + k*cos(theta2)
ry <- sin(theta1) + k*sin(theta2)
rx
ry

# need to load base package
theta3 <- atan2(ry, rx)
plot(rx, ry)

theta3 <- theta3*180/pi
mean(theta3)
sqrt(var(theta3))
hist(theta3)
```