Purpose:

This lab has four purposes:

- You will learn how to use simple Matlab codes to compute Midpoint, Trapezoidal, and Simpson approximation to definite integrals.
- You will use a couple of more sophisticated codes that are embedded in Matlab.
- You will be able to reproduce values of the cumulative normal standard distribution.
- You will discover how accurate this numeric estimation can be.

This lab uses the TI-82 program INTEGRAL, which you should already have downloaded from my calculator. Remember, to use this program, first put the appropriate function in Y1, then use program INTEGRAL.

Problem #1: Given following definite integral \( \int_{1}^{2} \frac{1}{x^2} \, dx \).

a) Find the exact value of the integral.

b) Use the Midpoint, Trapezoidal and Simpson rule for values of \( n = 10 \), \( n = 50 \) and \( n = 100 \) to compute estimations of the given integral.

c) Compute the error of each estimation considering all the digits given by your calculator.

To turn in for Problem #1: Fill in the table attached to the end of this lab, based on your work on parts (c) and (d).

Problem #2: Given the integral \( \int_{0}^{\pi/2} x^2 \sin(x) \, dx \). If you don't remember how, consult

a) Use the formula error for the Simpson rule to estimate the given integral with an error smaller than 0.000001. (To estimate the maximum value of the 4th derivative, you might want to first find the 4th derivative, then graph it on the interval shown.)

b) Use the Simpson rule to estimate the integral with the value of \( n \) you obtained from part (a).

To turn in for Problem #2: your work for part (a)—at least enough so that we can see how you got your value of \( n \)—and your estimate from (b). You should, obviously, carry enough decimal places to let us determine if your estimate meets the desired accuracy target.
Problem #3: The probability function of the standard normal distribution (the "bell curve") is given by \( f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \). The cumulative distribution function is defined by \( F(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx \). This cumulative distribution function, \( F(a) \), is arguably the single most important function in all of statistics, and therefore, it is vital to be able to evaluate it.

Unfortunately, we can't use the Fundamental Theorem of Calculus: there is no primitive function (antiderivative) for \( f(x) \). As a result, one option is to use a numerical method to evaluate the definite integrals. Because \( f(x) \) is an even function and the fact that \( F(\infty) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx = 1 \), then \( F(0) = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx = \frac{1}{2} \). This allows us to compute \( F(a) \) for \( a > 0 \) as follows \( F(a) = \frac{1}{2} + \int_{0}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx \). In a similar way you can compute \( F(a) \) for \( a < 0 \).

Because of the importance of \( F(a) \), statistics books have tables of values of \( F(a) \) for different values of \( a \). In a table for the cumulative normal standard distribution function the values for \( a = 1 \), is \( F(1) = 0.8413 \) and for \( a = 2 \), is \( F(2) = 0.9772 \). Can you reproduce these values using (for example) the Simpson rule? Repeat the steps (a) and (b) from the last problem to estimate \( F(1) \) and \( F(2) \) to at least the same accuracy as the table.

To turn in for Problem 3: same information as you did for Problem 2.