Ready for Lab 10 help

If you find difficult to get answers for ready for lab question study the following question which is very similar to the one in Ready for lab section of lab 10.

The Environmental Protection Agency recently investigated a spill of radioactive iodine. Measurements showed the ambient radiation levels at the site to be more than five r times the maximum acceptable limit of 0.6 millirems/hour (abbreviated mrems/hr). The EPA ordered an evacuation of the area. You will investigate several aspects of this incident using integrals and other basic information about functions.

At the beginning of the investigation, the emission rate was measured to be 3.2 mrems/hr. Over the first five hours, the following data were recorded:

<table>
<thead>
<tr>
<th>Time(Hrs)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation(mrems/Hr)</td>
<td>3.20000</td>
<td>3.18404</td>
<td>3.16816</td>
<td>3.15236</td>
<td>3.13664</td>
<td>3.12099</td>
</tr>
</tbody>
</table>

1. Based on the numbers in the table, calculate an upper and lower bound for the total amount of radiation (in mrems) emitted in the first five hours. (LRS and RRS)

Ans:
As you can see from the graph, the amount of radiation emitted decreases with time. At 0 Hrs it was 3.2 mrems and at 1 Hrs it is at 3.18404 mrems. If you really want to calculate the exact amount of radiation emitted during the first hour you may have to find a function that goes through all these five points and then integrate it from 0 to 1.
Exact amount of radiation emitted = \( \int_{0}^{1} R(t) \, dt \)

Here we want to calculate just an upper bound for total radiation emitted. So let us try something else. If we assume that the level of radiation emitted did not change during the first hour, then we can find an upper bound for radiation emitted. We can guarantee that this is an upper bound because we know that the level of radiation decreases with time, but here we have taken it as a constant.

Total amount of radiation 0-1 Hrs = 3.2 x 1 = 3.2

Using the same technique, we can assume that the amount of radiation emitted did not change during the second hour and we have:

Total amount of radiation 1-2 Hrs = 3.18404 x 1 = 3.18404

If we continue this method we have an upper bound for total radiation emitted during the first 5 hrs, that is:
3.2 + 3.18404 + 3.16816 + 3.15236 + 3.13664 = 15.84120

We can do the same thing to find a lower bound. During the first hour the level of radiation fell from 3.2 to 3.18404. What about if we assume that during the first hour the radiation level was at 3.18404 which is in fact was at a higher value and came down to this value at the end of the one hour period. This way we can guarantee that 3.18404 x 1 is a lower bound for total radiation emitted during the first hour.

Lower bound for total amount of radiation emitted during the first five hours would be:

3.18404 + 3.16816 + 3.15236 + 3.13664 + 3.12099 = 15.76219

We know one thing for sure, the exact amount of radiation emitted should be between 15.84120 merems/hrs and 15.76219 merems/hrs.

2. Let \( R(t) \) denote the radiation level in mrems/hr at time \( t \) hours after the iodine spill. Write an integral that represents the total amount of radiation (in mrems) emitted in the first five hours.
Well we already know the answer for this question. It is:

\[ \int_0^5 R(t) \, dt \]

3. If the EPA relied solely on measurements, it would have to keep measuring every hour to see when it was safe to return to the site of the spill. It would be better if there was a formula which predicted radiation levels at future times. There is! Radiation levels decay exponentially, so \( R(t) \) can be written as \( R_0 e^{kt} \). Find \( R_0 \) and \( k \) for the data in the table.

If that is the case, then:

\[ R(t) = R_0 e^{kt} \]

We know that \( R(0) = 3.2 \).

Therefore, \( 3.2 = R_0 e^{k(0)} \)

\[ 3.2 = R_0 \times 1 \quad \text{and} \quad R_0 = 3.2. \]

We also know that \( R(1) = 3.18404 \). (You don’t have to pick 1, you can pick any value from 1 – 5). Therefore,

\[ 3.18404 = R_0 e^{k(1)} \]

\[ 3.18404 = 3.2e^k \]

using Derive or any other method solve for \( k \) and you should get \( k = -0.005 \)

Then \( R(t) = 3.2e^{-0.005t} \)

4. Based on your formula from #3, when will the radiation level fall to the EPA’s estimate of an “acceptable” radiation level of 0.6 mrems/hr?

For certain value of \( t \), \( R(t) = 0.6 \) merems/Hrs. Therefore

\[ 0.6 = 3.2e^{-0.005t} \]

Once again using Derive or any other method solve for \( t \) and you should get \( t = 334.7952867 \) Hrs. That means it will take about 335 Hrs for the level of radiation to go down to 0.6 merems/Hrs.

5. Based on the data in the Table for radiation, is the left Riemann sum going to be an underestimate, or an overestimate? Why? is the right Riemann sum going to be an underestimate, or an overestimate? Why?

By now you should be able to find the answer for this question.

6. I will leave this question for you.

Start working on Ready for Lab 10 and use this question as a guide to get answers for those questions in the Ready for lab section.