A Network Condition-Centric Flow Selection and Rerouting Strategy to Mitigate Air Traffic Congestion under Uncertainties

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Rerouting is an effective air traffic flow management strategy to reduce en-route traffic congestion caused by convective weather or other off-nominal situations. In this paper, we introduce a network condition-centric flow selection and rerouting method to improve the efficiency of rerouting in an uncertain and dynamically changing airspace environment. The method adjusts the flows to be rerouted according to different network conditions to maximally reduce traffic congestion. To address the challenges such as high-dimensional demands and weather uncertainties, we adopt a scalable sampling-based control method that enables an efficient and robust optimal rerouting design. A series of analysis and simulation studies motivate and illustrate good performance of the proposed method.

I. Introduction

The growth of air traffic demands increases the burden of the National Airspace System (NAS), which has limited capacities. The strategic air traffic flow management (ATFM) aims to mitigate traffic congestion caused by the imbalance between capacities and demands at a long look-ahead time frame (2-15 hours in advance). Uncertain convective weather, which modulates the capacities of impacted regions, significantly complicates the decision process for ATFM at the strategic time frame. Other off-nominal situations such as security incidents have further challenged the effectiveness of strategic ATFM. These weather and other off-nominal situations may easily lead to long-lived traffic delays, incurring significant costs and traveler complaints. Efficient tools that can quickly generate effective strategic ATFM solutions to maximally reduce traffic congestion under uncertainties are urgently needed.

Typical strategic traffic management actions (called traffic management initiatives or TMIs) include ground delay programs (GDPs) that delay aircraft at the departure airports, miles and minutes-in-trails (MIT/MINIT) that delay aircraft in the air, rerouting that redistributes flows to avoid adverse situations, and airspace flow programs (AFPs) that assign estimated departure clearance times to aircraft scheduled to pass through a constrained airspace, etc. In this paper, we focus on rerouting that mitigates traffic congestion through changing flight routes.

Air traffic rerouting has been widely studied.1,3–11 In article,3 routes and airborne delays are assigned to each individual flight in the order determined by scheduled departure times, with alternative routes given in advance. Weather scenarios used for decision making are updated to capture the most recent forecasts. Article4 introduces a dynamic reroute generation algorithm to route the aircraft around the hazardous airspace. The route is updated when new weather forecast is received. Many other articles5–9 also consider reroute generation. Article10 introduces a network flow model to manage network flow at a larger spatial scale. Although aggregated flows are considered in the beginning, rerouting decisions are made for each individual flight, by applying a heuristic method to decompose aggregate flows into a set of flight paths, and selecting among them the near-optimal one for each individual flight. These individual flight-based

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approaches work well for traffic control at a shorter time scale, but can be inefficient in the strategic time scale when a large number of flights need to be scheduled in real time. Besides, as the performance of these approaches highly depends on the knowledge of particular deterministic weather forecasts, uncertainties and prediction errors of weather forecasts in the strategic time scale may easily lead to misleading management plans.

To address the aforementioned issues, flow-based models have been investigated to manage air traffic at the strategic time frame, and probabilistic weather information has been exploited to design TMIs that are robust to weather uncertainties. Managing traffic at an aggregated-flow level significantly reduces the computational costs, and permits planning at a broader scale. Article\textsuperscript{18} introduces a dynamic rerouting model which considers both GDP and rerouting to reduce total expected ground and airborne delays under uncertain weather. The optimal rerouting decisions are made for all flows that arrive at the same airport, and these decisions change over time in accordance with the updated probabilistic forecast of convective weather, described by a scenario tree. In our previous work,\textsuperscript{2} we developed a flow-based queuing network model to capture the dynamics of traffic flows under weather uncertainty and typical TMIs. This flow-based queuing network model is a critical component of the flow contingency management (FCM) framework\textsuperscript{19} for strategic ATFM. Based on this model, a sensitivity-based study was conducted in article\textsuperscript{20} to optimally reroute traffic flows at a particular divergence point under weather uncertainties.

In addition to the incapability of many existing rerouting strategies in addressing weather uncertainties and managing large flow volume (due to the use of individual flight-based models), these approaches often restrict the type of flows to be rerouted. They either only reroute flows that enter a constrained region, or directly redistribute all flights over the whole spatial scale. Through simulation studies, we found that rerouting the specific flows that pass a constrained region can mitigate traffic congestion at the region, but at a cost of higher demands and possible congestion in other regions. Planning rerouting over the whole spatial scale can solve the problem, but is expansive, especially for large and complicated networks. As such, it is critical to select proper range of flows to be rerouted.\textsuperscript{21}

In this paper, we develop a rerouting strategy based on the aggregated flow-based queuing network model\textsuperscript{2} to address the limitations of existing rerouting strategies, and efficiently and maximally reduce traffic congestion under demand and weather uncertainties. This rerouting approach adopts an innovative flow selection algorithm to proactively select a set of flows that need to be rerouted according to the network condition. This network condition-centric flow selection algorithm addresses the congestion propagation issue caused by rerouting, and also significantly reduces the computational cost required to redistribute all flows.

After the rerouting flows are selected, a stochastic optimal control problem is formulated to optimally redistribute selected flows to minimize an expected cost metric. In order to address high-dimensional weather and demand uncertainties, we apply a scalable uncertainty evaluation method called M-PCM-OFFD,\textsuperscript{22,23} which integrates multivariate collocation method (M-PCM) and orthogonal fractional factorial design (OFFD), to sample the uncertainty space. It is proved that a control solution optimal to the samples selected by M-PCM-OFFD is also optimal to the whole uncertainty space under mild assumptions. We use small examples to motivate and illustrate the features and procedures of the proposed approaches. A series of simulation and comparative studies are conducted to demonstrate capabilities of the proposed rerouting strategy in mitigating traffic congestion and improving computational efficiency.

The rest of this paper is organized as follows. In Section II, we show the preliminary results, including the aggregated flow-based queuing network model, and M-PCM-OFFD based stochastic optimal control method. In Section III, we discuss the limitations of existing rerouting strategies to motivate our study. In Section IV, we introduce the network condition-centric flow selection algorithm to address these limitations. In Section V, we describe the rerouting strategy to mitigate traffic congestion under demand and weather uncertainties. Section VI includes a brief conclusion.

\section*{II. Preliminaries}

In this section, we first briefly review the flow-based queuing network model,\textsuperscript{2} which serves as the evaluation and design foundation in this study. We then introduce the M-PCM-OFFD based stochastic optimal control method,\textsuperscript{24} which will be used to solve the stochastic optimal rerouting problem under high-dimensional demand and weather uncertainties.
II.A. Stochastic Modeling Framework for Air Traffic Flow Management

The stochastic flow-based queuing network model was developed in our previous study\textsuperscript{2} to facilitate the strategic ATFM,\textsuperscript{19} which captures air traffic flow dynamics under demand and weather uncertainties. In this model, the air traffic system is described by multiple overlapped sub-networks, differentiated by origin-destination (O-D) pairs. In each sub-network, flows from the origin airport (or airport group) travel through multiple routes to the destination airport (or airport group), with flow volumes determined by flow fractions. Figure 1 shows an example queuing network structure, which consists of two sub-networks. Three routes exist from origin 1 to destination 1, and two routes exist from origin 2 to destination 2. Please refer to article\textsuperscript{2} for detailed description of this queuing network structure.

![Figure 1. A simple queuing network structure consisting of two sub-networks differentiated by blue and orange colors.\textsuperscript{2}](image)

At a merging/splitting point $j$, the dynamics of traffic flows from origin $o$ to destination $d$ (indicated by $od$ in the subscripts) can be described by following mathematical equations:\textsuperscript{2}

\[
g_{odjl}[k] = \begin{cases} 
  p_{odjl} f_{odij}[k], & \text{if } j \in \theta_{od} \\
  \sum_l f_{odij}[k], & \text{else}
\end{cases}
\]

(1)

\[
\sum_l p_{odjl} = 1, \quad 0 \leq p_{odjl} \leq 1
\]

(2)

where $\theta_{od}$ denotes the full set of splitting points in sub-network $od \in \Omega$, where $\Omega$ is the full set of O-D pairs. $g_{odjl}[k]$ represents the number of aircraft leaving node $j$ and heading to node $l$ in sub-network $od$ at time $k$, $f_{odij}[k]$ describes the number of aircraft entering node $j$ along the link from node $i$ to node $j$ in sub-network $od$ at time $k$, and $p_{odjl}$ is the fraction of flows that leave node $j$ and head to node $l$ in sub-network $od$. As a special case, the dynamics of flows at origin $o$ can be described by $g_{odol}[k] = p_{odol} f_{od}[k]$, where $f_{od}[k]$ is the total demand from origin $o$ to destination $d$ at time $k$.

Flows arriving at the boundary of a region are considered to enter a virtual buffer located at the boundary intersection point, which is the intersection of region boundary and flow route.\textsuperscript{2} We here use $ijm$ to represent the boundary intersection of region $m$ and link $ij$, and use $u_{ijm}[k]$ to denote the number of incoming aircraft arriving at $ijm$ at time $k$. The number of aircraft that can leave the buffer (or boundary intersection point) and enter the region is described by $e_{ijm}[k]$, which is restricted by the capacity $N_{ijm}[k]$. Aircraft that are not allowed to enter the region is queued in the buffer, with its length captured by the backlog $b_{ijm}[k]$, an indicator of the severity of traffic congestion. The dynamics of flows at a boundary interaction point $ijm$
can be described by the following equations:

\[ e_{ijm}[k + 1] = \min(N_{ijm}[k + 1], b_{ijm}[k] + u_{ijm}[k + 1]) \]  
(3)

\[ b_{ijm}[k + 1] = \max(0, b_{ijm}[k] + u_{ijm}[k + 1] - N_{ijm}[k + 1]) \]  
(4)

\[ u_{ijm}[k] = \sum_{\forall od \in \Omega} g_{odij}[k - K_{i,ijm}] \]  
(5)

\[ N_{ijm}[k + 1] = \begin{cases}  
    b_{ijm}[k] & \text{if } b_{m}[k] > N_{m} \\
    b_{ijm}[k] + u_{ijm}[k + 1](N_{m} - b_{m}[k]) & \text{if } b_{m}[k] \leq N_{m}, u_{m}[k + 1] > 0 \\
    b_{ijm}[k] & \text{else}
\end{cases} \]  
(6)

where \( b_{m}[k] = \sum_{i,j} b_{ijm} \) and \( u_{m} = \sum_{i,j} u_{ijm} \) are the total backlog and total demand in region \( m \) at time \( k \), respectively. \( N_{m} \) is the capacity of region \( m \), and \( K_{i,ijm} \) is the number of time steps to travel from node \( i \) to boundary intersection point \( ijm \). We note that flow fraction \( p_{odij} \) captures the rerouting strategy, and \( N_{m} \) can model weather impact and flow-restriction TMIs such as MIT/MINIT.\(^2\) Furthermore, through restricting departure rates of flows at origins or arrival rates of flows at destinations, we are able to capture other TMIs such as GDP and AFP.\(^2\) Interested readers can refer to the article\(^2\) for a complete description of the queuing network model and its capabilities in capturing different TMIs. In this study, we focus on the rerouting strategy to ameliorate en-route congestion. We use \( N_{m} \) to model the impact of convective weather or other adverse situations that cause the reduction of region capacities.

### II.B. M-PCM-OFFD based Stochastic Optimal Control

The M-PCM-OFFD based stochastic optimal control method\(^{22,25}\) finds the optimal control solution at a reduced uncertainty space sampled by the M-PCM-OFFD.\(^{22,25}\) In this section, we first briefly describe M-PCM-OFFD, and then discuss the use of M-PCM-OFFD to facilitate stochastic optimal control.

#### 1. M-PCM-OFFD

The M-PCM-OFFD\(^{22,25}\) is a scalable and efficient uncertainty evaluation method which breaks the curse of dimensionality faced by the baseline method, M-PCM. It first utilizes the M-PCM to sample the uncertainty space according to the statistics of uncertain parameters, and then applies the OFFD to further reduce the uncertainty space. This integrated method has many nice features. It can not only efficiently and accurately predict the mean of system output under mild assumptions (see Lemma II.1), but is also robust to numerical errors.\(^{22,25}\)

**Lemma II.1.** Consider a multivariate system modulated by \( m \) uncertain parameters, where each parameter \( a_{i} \) has a degree up to 3 and follows an independent distribution. The system dynamics are described by the following system mapping

\[ Y(a_{1}, a_{2}, \ldots, a_{m}) = \sum_{j_{1}=0}^{3} \sum_{j_{2}=0}^{3} \cdots \sum_{j_{m}=0}^{3} \Phi_{j_{1},\ldots,j_{m}} \prod_{i=1}^{m} a_{i}^{j_{i}} \]  
(7)

Suppose high-order interactions are negligible, i.e., \( \Phi_{j_{1},\ldots,j_{m}} = 0 \) if more than \( \tau \) of \( j_{1}, \ldots, j_{m} \) are non-zero. The M-PCM-OFFD then approximates the original mapping with

\[ Y^{*}(a_{1}, a_{2}, \ldots, a_{m}) = \sum_{j_{1}=0}^{1} \sum_{j_{2}=0}^{1} \cdots \sum_{j_{m}=0}^{1} \Phi^{*}_{j_{1},\ldots,j_{m}} \prod_{i=1}^{m} a_{i}^{j_{i}} \]  
(8)

such that \( E[Y(a_{1}, a_{2}, \ldots, a_{m})] = E[Y^{*}(a_{1}, a_{2}, \ldots, a_{m})] \). \( \Phi^{*}_{j_{1},\ldots,j_{m}} = 0 \) if more than \( \tau \) of \( j_{1}, \ldots, j_{m} \) are non-zero.

In Algorithm 1, we sketch the key design steps of M-PCM-OFFD, which integrates M-PCM and \( 2^{m-\gamma} \) OFFD to reduce the number of simulations from \( 2^{m} \) to \( 2^{m-\gamma} \) in the range of \([2^{\lceil \log_{2}(m+1) \rceil}, 2^{m-1}]\), where \( \mathcal{R} \) defines the resolution of OFFD.\(^{26}\) Please refer to articles\(^{23,27}\) for procedures of M-PCM and articles\(^{22,25}\) for procedures of OFFD and detailed descriptions of M-PCM-OFFD.
over the planning horizon. Note that for each admissible control value \( p \) \( J \) space, the total expected cost
\[
\gamma^* = \max\{\gamma | 1 \leq \gamma \leq m - \lceil \log_2(\sum_{i=0}^{\gamma} a_i^\gamma) \rceil, \text{ and } 2^{m-\gamma} \text{ OFFD exists with } R \geq 2^\tau + 1 \};
\]
Apply \( 2^m \gamma^* \) OFFD to select \( \gamma^*_a \) simulation points from the full set of \( 2^m \) M-PCM
points.

end

Run simulation at each of the selected simulation points \((a_1, a_2, \ldots, a_m)_i\) to obtain the corresponding
output value \( Y(a_1, a_2, \ldots, a_m)_i \);

Given a set of inputs \((a_1, a_2, \ldots, a_m)_i\), and corresponding outputs \( Y(a_1, a_2, \ldots, a_m)_i \), solve Equation 8
to find the coefficients \( \Phi_{j_1, \ldots, j_m}^* \).

Algorithm 1: M-PCM-OFFD

1. Apply M-PCM to select \( 2^m \) \( m \)-tuple simulation points \((a_1, a_2, \ldots, a_m)_i \), \( i = 1, 2, \ldots, 2^m \);
   // Apply OFFD to further reduce the number of simulation points if possible
2. if \( m > 2 \) and \( 1 \leq \tau \leq \lceil \frac{m}{2} \rceil \) then
3. \[
\gamma^* = \max\{\gamma | 1 \leq \gamma \leq m - \lceil \log_2(\sum_{i=0}^{\gamma} a_i^\gamma) \rceil, \text{ and } 2^{m-\gamma} \text{ OFFD exists with } R \geq 2^\tau + 1 \};
\]
4. Apply \( 2^m \gamma^* \) OFFD to select \( \gamma^*_a \) simulation points from the full set of \( 2^m \) M-PCM
points;
5. end

2. Stochastic Optimal Control

Consider a dynamical system described by the following equation:
\[
x[k+1] = h_k(x, p, a), k = 1, \ldots, T - 1
\]
where \( x[k] \), \( p[k] \) and \( a[k] \) are the state, control and uncertain parameter vectors, respectively. \( T \) is
the terminal time instance describing the planning horizon. \( h_k(\cdot) \) describes the system dynamics at time \( k \).
The stochastic optimal control problem is concerned with finding the optimal control solution \( p^*[k] \) that
minimizes the following total expected cost:
\[
J_T = E_{a[1]} \left\{ \cdots E_{a[T-1]} \left\{ \sum_{k=1}^{T-1} \alpha^k y_k(x, p, a) + q_T(x) \right\} \cdots \right\}
\]
where \( y_k(\cdot) \) and \( q_k(\cdot) \) are cost functions at time \( k \), and \( \alpha \in (0, 1) \) is a discount factor.

The key idea of the M-PCM-OFFD based control approach is to use M-PCM-OFFD to sample the
uncertainty space and estimate the expected costs. Consider the case where \( a[k] \) and \( p[k] \) are time-invariant
over the planning horizon. Note that for each admissible control value \( p \in C \), where \( C \) defines the control
space, the total expected cost \( J_T \) can be considered as the mean output of a system of the following system
mapping
\[
Y(a) = \sum_{k=1}^{T-1} \alpha^k y_k(x, p, a) + q_T(x).
\]
Therefore, we can apply M-PCM-OFFD to approximate the mean output in an efficient way. In particular, if
the degree of each of the uncertain parameters \( a_i \) does not exceed 3 and the high-order interactions involving
more than \( \tau \) parameters are negligible, the M-PCM-OFFD can accurately estimate the total expected cost
\( J_T = E_a[Y(a)] \) for each \( p \in C \). The optimal control solution \( p^* \) with the lowest total expected cost can
then be found by evaluating all possible control solutions. We have proved in article\textsuperscript{24} that the control
solution optimal to the samples selected by M-PCM-OFFD is also optimal to all possible values of uncertain
parameters under aforementioned assumptions. For systems with time-varying \( a[k] \) and \( p[k] \), dynamical
programming\textsuperscript{28} or reinforcement learning\textsuperscript{29} can be used to find the optimal control solution under the
reduced uncertainty space sampled by M-PCM-OFFD. Interested readers can refer to article\textsuperscript{24} for detailed
descriptions of the M-PCM-OFFD based stochastic optimal control.

III. Motivation

In this section, we investigate the limitations of existing rerouting strategies, which motivate the network
cost, and congestion method. Existing rerouting strategies mitigate traffic congestion caused by region capacity reduction mainly in two ways: 1) rerouting flows entering the capacity-reduced region or 2) redistributing all flows in the network. The first strategy is computational efficient but may cause congestion in other regions. The second one can maximally reduce the traffic congestion but is not
scalable to the network size and is very computationally expansive for large and complicated networks. Let us next use an example to illustrate potential problems of the two traditional rerouting strategies.

Consider a small-scale air traffic network shown in Figure 2, where the airspace is composed of four sectors (regions): $S_4$, $S_5$, $S_6$, $S_7$. The dashed lines represent region boundaries. This network contains three origin airports (nodes 1, 2, and 3), two destination airports (nodes 20 and 21) and five O-D pairs (1-20, 1-21, 2-20, 2-21, and 3-20). Under nominal operations, the capacities of sectors $S_4$, $S_5$, $S_6$, and $S_7$ are 5, 4, 3, 2 aircraft per time interval, respectively.

![Figure 2. A small-scale air traffic network with five O-D pairs. Directed lines represent the routes, with each line associated with a travel time measured in hours (purple numbers) and flow fraction (red numbers). Flows in different O-D sub-networks are differentiated by different line colors.](image-url)

Suppose sector $S_5$ is disabled (i.e., $N_{S5} = 0$) for a long period of time, due to long-lived convective weather or other off-nominal events. In order to reduce traffic congestion (measured by total backlog), the first rerouting strategy reroutes aircraft entering region $S_5$ to other regions, by modifying flow fractions $p = \{p_{2,20,1.5}, p_{2,20,10,9}, p_{1,20,5,13}, p_{2,20,2,10}, p_{2,20,10,8}, p_{1,20,5,16}\}$. Performance of this rerouting design was investigated in the article. The second rerouting strategy redistributes all flows by modifying the full set of flow fractions $p = \{p_{odij}\}$. The optimal flow fractions $p^*$ can then be found by solving the following optimization problem:

$$\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{T} B[k] \\
\text{subject to} & \quad \sum_{j} p_{odij} = 1, \ 0 \leq p_{odij} \leq 1, \ p_{odij} \in p
\end{align*}$$

(12)

where $B[k] = \sum_{n} b_{nm}[k]$ is the total backlog at time step $k$. To investigate the performance of the two strategies under different network conditions, we vary the traffic demand in sub-network 2-20, i.e., $f_{2,20}[k]$, and set $f_{od}[k] = 2$ for all other O-D pairs. In this example and following studies, we assume the demands at all departure airports are constant and time-invariant during the planning horizon. The planning horizon of $6h$ is divided into $T = 24$ time steps, with the time interval set to $15\text{min}$.

The simulation results are shown in Figure 3. As we can see, the performance of the first rerouting strategy that reroutes only flows entering $S_5$ degrades dramatically when the departure rate $f_{2,20}$ exceeds 2, as reflected by the fast growth of the total backlog. This is because rerouted flows cause the increase of demands in other regions, as such lead to capacity-demand imbalance. The second rerouting strategy alleviates this issue, by redistributing all flows. However, solving the optimization problem in Equation 12 with a full set of flow fraction parameters (14 in this example) to be determined is time-consuming, especially for large and complicated networks with large number of flow fraction parameters. Another observation we
can obtain from Figure 3 is that when the departure rate is relatively small (smaller than 1.5), the total backlog remains zero for both rerouting strategies, indicating the dispensability of redistributing all flows.

Motivated by this observation and limitations of existing rerouting strategies, we explore in the following section an innovative approach to automatically select flows that need to be rerouted.

IV. Network Condition-Centric Flow Selection Algorithm

In this section, we first describe the network condition-centric flow selection algorithm that addresses limitations of the two traditional rerouting strategies discussed above. Simple examples are used to illustrate design ideas and procedures of the proposed approach. A comparison study is then conducted to demonstrate the flow selection algorithm.

IV.A. Algorithm Description

The network condition-centric flow selection algorithm consists of two phases: 1) initial search phase to find flows that directly lead to traffic congestion; and 2) amelioration of congestion propagation phase to find additional flows that can ameliorate congestion propagation if rerouted. Let us now describe each phase.

1. Initial Search

This phase aims to find flows that directly cause capacity-demand imbalance in constrained regions. According to Equations 4 and 6, the total backlog can be described by the following equations:

\[
B[k+1] = \sum_m b_m[k+1] = \sum_m \sum_{i,j} \max(0, b_{ijm}[k] + u_{ijm}[k + 1] - N_{ijm}[k+1])
\]

\[
= \sum_m \max(0, b_m[k] + u_m[k + 1] - N_m)
\]

where \(b_m[0] = 0\). Therefore, if \(u_m[k] \leq N_m\) for each region \(m\) at each time step from 0, we will have \(B[k] = 0\). Otherwise, if \(u_m[k] > N_m\), we have \(B[k] > 0\). From this analysis, we can infer that flows \(g = \{g_{odi}\}\) entering region \(m\) will cause traffic congestion if the following condition is satisfied:

\[
u_m[k] = \sum_{i,j} \sum_{\forall \text{od} \in \Omega} g_{odi}[k - K_{i,ijm}] > N_m,
\]

As such, this set of flows need to be rerouted. To perform rerouting, we need to find associated flow fractions \(p\) that control the volumes of flows \(g\). The procedures to determine \(p\) given \(g\) are summarized in Algorithm 2.
Algorithm 2: $p = \text{findFraction}(g)$

1. 
   foreach $g_{odi}j \in g$
   2.     if $i \in \theta_{od}$ then
   3.         $p \leftarrow p \cup \{p_{odi}j\}$, where $\sum_j p_{odi}j = 1$;
   4.     else
   5.         Find the splitting point $l \in \theta_{od}$ closest to node $i$ among its previous hops;
   6.     $p \leftarrow p \cup \{p_{odi}j\}$, where $\sum_j p_{odi}j = 1$;
   7.   end

Example IV.1. We use the air traffic network described in Section III to illustrate procedures of the initial search phase. Suppose the departure rate $f_{2,20} = 2$. As region $S5$ is impacted by convective weather and $N_{S5} = 0$, we calculate the total demand $u_{S5}^*\{k\}$ and compare it with capacity $N_{S5}$. In particular, $u_{S5}^*\{k\} = 2$ and $u_{S5}^*\{k\} > N_{S5}$. Therefore, flows $g = \{g_{2,20,10,9}, g_{2,20,2,15}, g_{1,20,5,13}\}$ that enter region $S5$ need to be rerouted. We then follow Algorithm 2 to find corresponding flow fractions that control the volumes of flows crossing both regions. The equality holds when flows are only redistributed over regions that do not cross both regions. We then follow Algorithm 2 to find corresponding flow fractions that control the volumes of flows rerouted. We then follow Algorithm 2 to find corresponding flow fractions that control the volumes of flows crossing both regions.

Example IV.2. Let us continue using Example IV.1 to illustrate the congestion propagation issue. By solving the optimization problem in Equation 12, we can obtain the optimal flow fractions $p^*$ that minimize the total backlog, i.e., $p^*_{2,20,10,9} = p^*_{2,20,2,15} = p^*_{1,20,5,13} = 0$ and $p^*_{2,20,10,8} = p^*_{2,20,2,10} = p^*_{1,20,5,16} = 1$, which make all flows entering region $S5$ be rerouted to other regions ($S4$ and $S6$). After rerouting, the total demands in regions $S4$ and $S6$ increase to $u_{S4}^*\{k\} = 4$ and $u_{S6}^*\{k\} = 3.4$, respectively. As $N_{S6} = 3 < u_{S6}^*\{k\}$, backlogs are generated in region $S6$.

In Example IV.2, the backlogs in region $S6$ can be eliminated by further rerouting flows entering region $S6$. Motivated by this example, we develop an approach to predict and ameliorate the congestion propagation. The key idea is to further reroute flows entering regions that are likely to be congested after rerouting. Suppose region $m$ is impacted by convective weather, and a set of flows $g$ to be rerouted and corresponding flow fraction parameters $p$ are selected by performing the initial search. Define $I_m$, $m \not\in I_m$, as the full set of regions along the alternative paths of flows $g$, whose demands may increase if $g$ are rerouted. We then aim to evaluate the impact of rerouting flows $g$ on regions $I_m$. Note that the optimal rerouting solution $p^*$ is found by minimizing the total backlog (see Equation 12). Therefore, if an admissible rerouting solution $p$ exists that will not cause congestion in region $i \in I_m$, then no congestion will occur when the optimal solution is applied. Motivated by this idea, we develop the following approach.

Suppose rerouting will cause an increment of demand $\Delta u_i^*\{k\}$ in each region $i \in I_m$, which is unknown. Instead of directly estimating $\Delta u_i^*\{k\}$, we evaluate the possible total demand in region $i$ combining with region $m$ after rerouting, as an increment of demand $\Delta u_i^*\{k\}$ in region $i$ corresponds to a decrement of the same demand volume in region $m$. In particular, denoting the total demand of the combined region $i,m$ after optimal rerouting as $u_{i,m}^*\{k\}$, we can then easily derive $u_{i,m}^*\{k\} \leq u_i^*\{k\} + u_m^*\{k\} - \Delta$, where $\Delta$ is the volume of flows that pass both regions $i$ and $m$, and can be rerouted by modifying $p$ to alternative paths that do not cross both regions. The equality holds when flows are only redistributed over regions $i$ and $m$ and flows crossing both regions are rerouted if alternative paths that only pass region $i$ or $m$ exist. Therefore, if $u_i^*\{k\} + u_m^*\{k\} - \Delta \leq N_i + N_m$, no congestion will occur. Otherwise, congestion may happen and thus flows entering region $i$ are selected to be rerouted, in which case we further evaluate the impact of rerouting these flows on regions $I_i$ along their alternative paths, by calculating $u_{i,m,j}^*\{k\}$ for each $j \in I_i$ and comparing it with $N_i + N_m + N_j$. This process continues until no new flows need to be rerouted.

Algorithm 3 summarizes the procedures of the amelioration of congestion propagation phase. This recursive function takes three inputs: 1) a set of regions $I$ to be evaluated, 2) flow fraction parameters $p$ that
have been selected, and 3) all regions $I'$ that are possible to be congested after rerouting.

**Algorithm 3:** For a set of regions $I$, the complete network condition-centric flow selection algorithm is shown in Algorithm 3, as discussed in Example IV.1. We thus set $I = \{S4, S6\}$ and $I' = \{S5\}$. With $p$ obtained from Example IV.1, we then follow procedures of Algorithm 3 to ameliorate congestion propagation. In particular, when $m = S6$, we have $u_{S6\&S5}[k] = g_{1,20,1.5} + g_{2,20,2.15} + g_{2,20,10.8} + g_{2,20,10.9} = 3.4$ and $N_{S6} + N_{S5} = 3$ (Lines 2-3). Therefore, $u_{S6\&S5}[k] > N_{S6} + N_{S5}$ and flow $g_{1,20,1.5}$ that enters region $S6$ is selected to be rerouted (Line 4). The corresponding flow fraction parameters are $\bar{p}_t = \{p_{1,20,1.4}, p_{1,20,1.5}\}$ (Line 5). As flow $g_{1,20,1.5}$ will be rerouted to region $S7$, we have $I_{S6} = S7$ (Line 7). $I' = \{S5, S6\}$ (Line 8). We then repeat above procedures to evaluate the impact of rerouting on region $S7$ (Lines 10-12). As $u_{S7\&S5\&S6}[k] = N_{S5} + N_{S6} + N_{S7} = 5$, no congestion will occur in region $S7$. We then evaluate region $m = S4$. As $u_{S4\&S5} = 4.7$ and $N_{S4} + N_{S5} = 5 > u_{S4\&S5}$, no new flows need to be rerouted. Therefore, the program is terminated and outputs $p = \{p_{1,20,1.8}, p_{2,20,10.1}, p_{2,20,2.9}, p_{1,20,5.1}\}$, where $s \in \{4, 5\}$, $i \in \{8, 9\}$, $j \in \{10, 15\}$ and $l \in \{13, 16\}$.

The procedures of the complete network condition-centric flow selection algorithm is shown in Algorithm 4, which requires three inputs: 1) a set of regions $M$ that are impacted by convective weather, 2) demand in each sub-network $\{f_{od}\}$ and 3) capacity of each region $\{N_i\}$. The output is a set of flow fraction parameters to be determined.

**Algorithm 4:** Network Condition-Centric Flow Selection Algorithm

<table>
<thead>
<tr>
<th>Input: $M$, ${f_{od}}$, ${N_i}$</th>
<th>Output: $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>foreach $m \in M$ do</td>
<td></td>
</tr>
<tr>
<td>// Initial Search</td>
<td></td>
</tr>
<tr>
<td>$u_m[k] = \sum_i \sum_{\ell \in \Omega} g_{o_dij}[k - K_{i,j,m}]$;</td>
<td></td>
</tr>
<tr>
<td>if $u_m[k] &gt; N_m$ then</td>
<td></td>
</tr>
<tr>
<td>// Find the full set of flows $g = {g_{o_dij}}$ that enter region $m$;</td>
<td></td>
</tr>
<tr>
<td>$p_t = \text{findFraction}(g);$</td>
<td></td>
</tr>
<tr>
<td>$p = p \cup p_t;$</td>
<td></td>
</tr>
<tr>
<td>// Amelioration of Congestion Propagation</td>
<td></td>
</tr>
<tr>
<td>Find the full set of regions $I_m$ along all alternative routes of flows $g$, where $m \notin I_m$;</td>
<td></td>
</tr>
<tr>
<td>foreach $i \in I_m$ do</td>
<td></td>
</tr>
<tr>
<td>// p ← congestionAmelioration($I_m, p, I'$);</td>
<td></td>
</tr>
<tr>
<td>end</td>
<td></td>
</tr>
<tr>
<td>end</td>
<td></td>
</tr>
</tbody>
</table>
IV.B. Comparative Studies

In this section, we investigate capabilities of the proposed network condition-centric flow selection algorithm in facilitating congestion mitigation through comparative studies.

Consider the air traffic network described in Section III, with two regions $S_4$ and $S_5$ impacted by convective weather. The capacities of $S_4$ and $S_5$ reduce to $N_{S_4} = 3$ and $N_{S_5} = 2$, respectively. We compare our approach with the two traditional rerouting strategies described in Section III.

The first traditional rerouting strategy reroutes all flows entering capacity-reduced regions ($S_4$ and $S_5$), by modifying $p = \{p_{1,21,1,1}, p_{1,22,1,2}, p_{2,20,10,13}, p_{2,20,2,4}, p_{1,20,5,16}\}$, where $i_1 \in \{6, 7\}$, $i_2 \in \{10, 12\}$, $i_3 \in \{8, 9\}$, $i_4 \in \{10, 15\}$, and $i_5 \in \{13, 16\}$. Therefore, it has 10 flow fraction parameters to be determined. The second traditional rerouting strategy reroutes all flows by modifying the full set of flow fractions, i.e., 14 flow fraction parameters. Different from these approaches, our approach selects flows to be rerouted based on the network condition. As shown in Figure 4(a), the number of flow fraction parameters selected by our approach varies for different network conditions. Figure 4(b) shows the total backlogs $\sum_{t=1}^{T} B[k]$ after applying different rerouting strategies, which are obtained by solving Equation 12.

The two figures illustrate the adaptability of our approach to different network conditions, which allows traffic congestion to be maximally reduced with less computational costs. Specifically, when $f_{2,20}$ is small (less than 1), no flows need to be rerouted as no backlogs will be generated (see red line in Figure 4(b)). As $f_{2,20}$ increases, congestion occurs in weather-impacted regions. Flows that enter these regions then need to be rerouted. When $f_{2,20}$ rises to 5, rerouted flows cause congestion in other regions (indicated by the blue line in Figure 4(b)) and thus more flows need to be rerouted. When $f_{2,20}$ is very large (larger than 5.5), the whole network is saturated.

![Figure 4](image)

Figure 4. Comparison of our approach with two traditional rerouting strategies in terms of a) number of flow fraction parameters selected, and b) performance in mitigating traffic congestion.

V. Rerouting under Demand and Weather Uncertainties

In this section, we discuss the rerouting design under demand and weather uncertainties. We first formulate the problem and then introduce the M-PCM-OFFD based optimal control to find the optimal rerouting solution that minimizes traffic congestion and is robust to demand and weather uncertainties. Simulation studies are then conducted to illustrate the performance of proposed approach.

V.A. Problem Formulation

Consider an air traffic network with dynamics described by the flow-based queuing network model introduced in Section II.A. The demand injected into each O-D sub-network, $f_{od}$, is modeled as a random variable, following an independent probability distribution, which can be derived from historical data. A set of regions $M$ are impacted by convective weather, the impact of which can be naturally modeled as capacity reduction. We here use a random variable to model the capacity of region $m \in M$, $N_m$, which follows
an independent probability distribution derived from weather forecasts. We assume all uncertain variables are time-invariant over the planning horizon. Based on this model, we aim to design the optimal rerouting scheme that minimizes a total expected cost \( J_T \). The stochastic optimal rerouting problem is formulated as follows

\[
\begin{align*}
\text{minimize} & \quad J_T = E_a[Y(a, p)] \\
\text{subject to} & \quad \sum_j p_{odi,j} = 1, \quad 0 \leq p_{odi,j} \leq 1, \quad p_{odi,j} \in p
\end{align*}
\]

(16)

where \( a \) denotes the full set of uncertain variables including \( \{f_{odi}\} \) and \( \{N_m\} \), \( m \in M \). \( Y(a, p) \) is the cost function.

V.B. M-PCM-OFFD based Optimal Rerouting

To address the stochastic optimal rerouting problem in Equation 16, we first apply the network condition-centric flow selection algorithm described in Section IV to determine the flow fraction (or control) parameters \( p \). Note that the flow selection algorithm requires the knowledge of traffic demand \( f_{odi} \) injected into each sub-network and the capacity \( N_i \) of each region. However, all \( f_{odi} \) and \( N_i \), \( m \in M \), are uncertain. To address this issue, the most conservative way is to take the maximum value of \( f_{odi} \) (denoted as \( f_{odi}^{max} \)), and minimum value of \( N_m \) (denoted as \( N_m^{min} \)) to find the maximum number of flow fraction parameters required for rerouting. This approach can maximally reduce traffic congestion, but is expansive. On the other hand, the most risky way is to take the minimum value of \( f_{odi} \) (denoted as \( f_{odi}^{min} \)) and maximum value of \( N_m \) (denoted as \( N_m^{max} \)) to find the minimum number of flow fraction parameters. This approach is more efficient but may perform poorly in congestion mitigation. In this study, we choose a less conservative but more efficient way, that is use mean values of \( f_{odi} \) and \( N_m \), denoted as \( \bar{f}_{odi} \) and \( \bar{N}_m \) respectively, to perform the flow selection algorithm.

Once the flow fraction parameters \( p \) are selected, we then apply the M-PCM-OFFD based stochastic optimal control method introduced in Section II.B to find the optimal control solution \( \mathbf{p}^* \). In particular, for each admissible value of \( \mathbf{p} \), we apply M-PCM-OFFD to estimate the output mean of \( Y(a, \mathbf{p}) \). \( \mathbf{p}^* \) can then be found, which has the minimal expected cost. \( \mathbf{p}^* \) is updated with new predictions of demand and weather statistics.

V.C. Simulation Studies

In this section, we investigate the performance of proposed rerouting strategy through simulation studies. Consider the small-scale air traffic network described in Section III. Suppose region \( S5 \) is impacted by uncertain convective weather, whose capacity is described by an uncertain variable that follows a uniform distribution \( N_{S5} \sim U(1,3) \). Assume demand \( f_{odi} \) in each sub-network also follows a uniform distribution, and \( f_{1,20} \sim U(1,3), \ f_{1,21} \sim U(1,1.5), \ f_{2,20} \sim U(2,3), \ f_{2,21} \sim U(1,2), \ f_{3,20} \sim U(0.5,1) \).

In the first experiment, we investigate the impact of using different demand and capacity values as inputs to the flow selection algorithm on the network performance. The objective cost function is set to \( Y(a, p) = \sum_{k=1}^T B[k] \). The minimum expected cost \( J_T^\tau \) is found by applying the M-PCM-OFFD (with \( \tau = 1 \)) based control approach. Table 1 summarizes the comparison results. As expected, the most conservative approach has the best performance in reducing traffic congestion, but has large number of flow fraction parameters to be determined. The most risky approach is the most efficient but generates the largest congestion cost. Our approach trades off between efficiency and congestion mitigation performance.

<table>
<thead>
<tr>
<th>Inputs of Flow Selection Algorithm</th>
<th>Number of Flow Fraction Parameters Selected</th>
<th>( J_T^\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{odi}^{max} ) and ( N_m^{min} ) (Most Conservative)</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>( f_{odi}^{min} ) and ( N_m^{max} ) (Most Risky)</td>
<td>6</td>
<td>52.4967</td>
</tr>
<tr>
<td>( \bar{f}_{odi} ) and ( \bar{N}_m ) (Our Approach)</td>
<td>6</td>
<td>3.8811</td>
</tr>
</tbody>
</table>

Table 1. Impact of using different values of inputs to the flow selection algorithm.
In the second experiment, we illustrate the performance of M-PCM-OFFD based approach in estimating the total expected cost and finding the optimal solution. In this experiment, we consider a more complicated cost function that includes both congestion and travel costs, i.e.,

$$Y(a, p) = \beta_1 \sum_{k=1}^{T} B[k] + \beta_2 \sum_{k=1}^{T} \sum_{odi} g_{odi} [k] c_{odi}$$

(17)

where $c_{odi}$ is the travel time for route segment $odi$ measured in hour. $\beta_1$ and $\beta_2$ are the weights. In this experiment, we set $\beta_1 = 1$, $\beta_2 = 0$.1 and extend the planning horizon to $T = 48$. The mean values of $f_{odi}$ and $N_m$ are used to run the flow selection algorithm, which selects 6 flow fraction parameters $p = \{p_{2,20,10,9} , p_{1,20,5,i_2} , p_{2,20,2,i_3}\}$, where $i_1 = \{8,9\}$, $i_2 = \{13,16\}$ and $i_3 = \{10,15\}$. We then apply M-PCM-OFFD based approach to find the optimal rerouting solution. With $\tau = 1$, the M-PCM-OFFD picks 8 samples to estimate the expected cost. The optimal rerouting solution found by this approach is $p_2^* = p_{2,20,10,9} = 0.3$, $p_{2,20,10,8} = p_{2,20,2} = 0.7$, $p_{1,20,5,13} = 0$ and $p_{1,20,5,16} = 1$, and the minimum total expected cost is $J_T^* = 289.9783$.

For comparison, we also apply the most widely used Monte Carlo (MC) simulation method to sample the uncertainty space. With the number of samples set to 5000, the MC based approach finds the same optimal rerouting solution. Figure 5 shows the performance of MC based approach in estimating the minimum total expected cost $J_T^*$, which requires large number of samples to converge to a meaningful value. However, the M-PCM-OFFD based approach only needs 8 samples to obtain a good estimation. Note that the minimum total expected cost $J_T^*$ estimated by MC based approach is slightly larger than the one obtained by the M-PCM-OFFD based approach. One possible reason is that high-order flow interactions may have significant impacts on air traffic dynamics and M-PCM-OFFD with $\tau = 1$ that ignores all interactions cannot capture flow dynamics well. This can be addressed by increasing the value of $\tau$.

![Figure 5](image.png)

**Figure 5.** Performance of the MC based method in estimating the minimum total expected cost $J_T^*$.

### VI. Conclusion

In this paper, we develop an optimal rerouting method which integrates a network condition-centric flow selection algorithm which proactively determines flows that need to be rerouted, and a scalable M-PCM-OFFD based optimal control method which determines the optimal rerouting solutions under high-dimensional demand and weather uncertainties. This method can automatically adjust the flows to be rerouted at different network conditions to maximally reduce traffic congestion. It can effectively ameliorate congestion propagation caused by rerouting, and improves the computational efficiency of rerouting design. A series of examples are used to motivate and illustrate the proposed approaches. Comprehensive comparative simulation studies show good performance of proposed rerouting strategy in terms of adaptability to varying network conditions, efficiency, and capability in mitigating traffic congestion.
Acknowledgments

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References


